Problem 2

I. Answer the following questions regarding transmission line circuits. Voltage and current are expressed by complex notation, the angular frequency is $\omega$, and the imaginary unit is $j$. The circuits are in a stationary state.

1. Fig. 1 shows the equivalent circuit of a lossless transmission line with an inductance per unit length of $L$ and a capacitance per unit length of $C$.

1-i) Find the complex impedances of the inductor and the capacitor in the small area enclosed by the dotted line between the position $x$ and $x + dx$.

1-ii) The current and the voltage at the position $x$ are $I$ and $V$, respectively. The current and the voltage at the position $x + dx$ are $I + dI$ and $V + dV$, respectively. Write down two first-order differential equations with respect to $x$, which hold between $I$ and $V$. The equations should include $\omega$.

1-iii) Write down the second-order differential equations with respect to $x$ for $I$ and $V$, respectively.

1-iv) The general solutions of the second-order differential equations in Question (1-iii) are written as follows:

\[
V = Ae^{-\gamma x} + Be^{\gamma x}
\]

\[
I = \frac{Ae^{-\gamma x} - Be^{\gamma x}}{Z_0},
\]

where $A$ and $B$ are constants. Express $\gamma$ and $Z_0$ using $\omega$, $L$, and $C$.

2. The transmission line is terminated by a resistance $Z_R$ at $x = l$ as shown in Fig. 2. The current through and the voltage across the resistor are $I_R$ and $V_R$, respectively. $\beta$ is defined as $\omega \sqrt{LC}$.

2-i) The first term and the second term of both Eq. (i) and Eq. (ii) represent a forward wave and a reflected wave, respectively. Find the relationship between $Z_R$ and $Z_0$, when $B$ becomes zero and thus no reflection occurs at $x = l$.

2-ii) When an input voltage is $V_S$ and an input current is $I_S$ at the input terminal $x = 0$, the complex impedance of the transmission line seen from the input is $Z_S = V_S/I_S$. Express $Z_S$ in the form as shown below. You may use $\tanh(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}$.

\[
Z_S = Z_0 \frac{\Box + \Box \tanh(\Box)}{\Box + \Box \tanh(\Box)}
\]
II. Answer the following questions regarding a circuit with an N-type MOS transistor.

A voltage between the gate and source of the transistor is $V_{GS}$. The transistor has a threshold voltage $V_{TH}$, a drain current $I_D$, and a transconductance $g_m$. For several values of $V_{GS}$, DC drain voltage – drain current characteristics are shown in Fig. 3. The drain voltage at the inflection point is $V_{GS} - V_{TH}$ and the drain current at the point is proportional to $(V_{GS} - V_{TH})^2$. A small-signal equivalent circuit of the transistor is represented by Fig. 4.

1. Express $g_m$ by using $V_{GS}$, $I_D$, and $V_{TH}$ when the transistor operates in the saturation region.

A voltage-amplifier circuit that consists of a transistor $M$, resistors, and capacitors as shown in Fig. 5. The supply voltage is $V_{DD}$. Small-signal input and output voltages are $v_{in}$ and $v_{out}$, and their Laplace transforms are $V_{in}(s)$ and $V_{out}(s)$, respectively. Here, $s$ is a variable of the Laplace transform.

2. A drain current $I_D$ flows through $M$ when the supply voltage $V_{DD}$ is applied. Then, find the maximum value of $R_L$ when $M$ operates in the saturation region, by using $V_{DD}$, $V_{TH}$, and $I_D$.

3. When $M$ operates in the saturation region, draw a small-signal equivalent circuit of the circuit in Fig. 5.

4. When $M$ operates in the saturation region, find a transfer function $\frac{V_{out}(s)}{V_{in}(s)}$.

5. Draw a Bode diagram of the transfer function in Question (4) with respect to the amplitude and the phase. Here, $C_1R$ is sufficiently larger than $C_2R_L$.

6. Find the angular frequency where the amplitude of the transfer function in Question (4) becomes unity at sufficiently high frequency.