Problem 2

I. Answer the following questions regarding transmission line circuits. Voltage and current are expressed by complex notation, the angular frequency is ω, and the imaginary unit is j. The circuits are in a stationary state.

(1) Fig. 1 shows the equivalent circuit of a lossless transmission line with an inductance per unit length of L and a capacitance per unit length of C.

(1-i) Find the complex impedances of the inductor and the capacitor in the small area enclosed by the dotted line between the position x and $x + dx$.

(1-ii) The current and the voltage at the position x are I and V, respectively. The current and the voltage at the position $x + dx$ are $I + dI$ and $V + dV$, respectively. Write down two first-order differential equations with respect to x, which hold between I and V. The equations should include ω.

(1-iii) Write down the second-order differential equations with respect to x for I and V, respectively.

(1-iv) The general solutions of the second-order differential equations in Question (1-iii) are written as follows:

$$V = Ae^{-\gamma x} + Be^{\gamma x} \qquad \text{(i)}$$

$$I = \frac{Ae^{-\gamma x}}{Z_0} - \frac{Be^{\gamma x}}{Z_0}, \qquad \text{(ii)}$$

where A and B are constants. Express γ and Z_0 using ω, L, and C.

(2) The transmission line is terminated by a resistance Z_R at $x = l$ as shown in Fig. 2. The current through and the voltage across the resistor are I_R and V_R, respectively. β is defined as $\omega\sqrt{LC}$.

(2-i) The first term and the second term of both Eq. (i) and Eq. (ii) represent a forward wave and a reflected wave, respectively. Find the relationship between Z_R and Z_0, when B becomes zero and thus no reflection occurs at $x = l$.

(2-ii) When an input voltage is V_S and an input current is I_S at the input terminal $x = 0$, the complex impedance of the transmission line seen from the input is $Z_S = V_S/I_S$. Express Z_S in the form as shown below. You may use $\tanh(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}$.

$$Z_S = Z_0 \frac{\square + \square\tanh(\square)}{\square + \square\tanh(\square)}$$

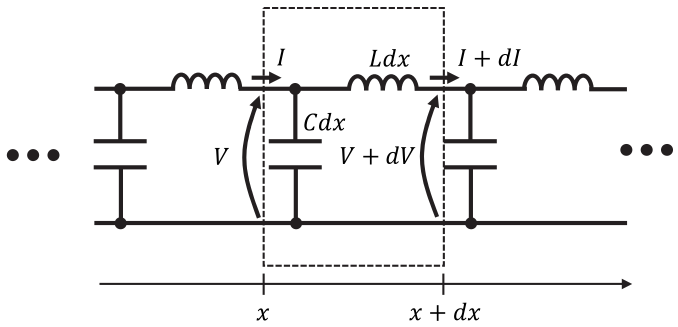


Fig. 1

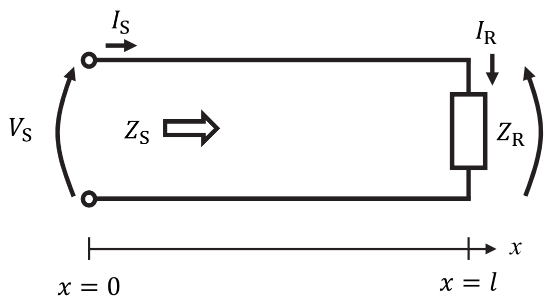


Fig. 2

II. Answer the following questions regarding a circuit with an N-type MOS transistor.

A voltage between the gate and source of the transistor is V_{GS}. The transistor has a threshold voltage V_{TH}, a drain current I_{D}, and a transconductance g_{m}. For several values of V_{GS}, DC drain voltage – drain current characteristics are shown in Fig. 3. The drain voltage at the inflection point is $V_{\mathrm{GS}} - V_{\mathrm{TH}}$ and the drain current at the point is proportional to $(V_{\mathrm{GS}} - V_{\mathrm{TH}})^2$. A small-signal equivalent circuit of the transistor is represented by Fig. 4.

(1) Express g_{m} by using V_{GS}, I_{D}, and V_{TH} when the transistor operates in the saturation region.

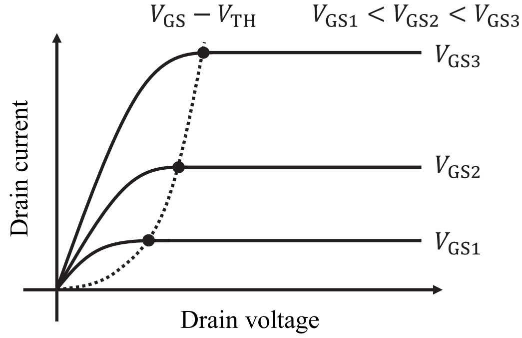


Fig. 3

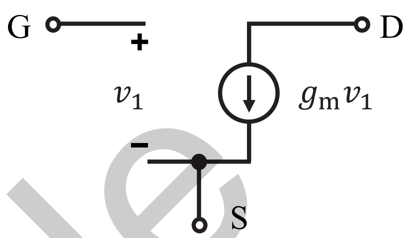


Fig. 4

A voltage-amplifier circuit that consists of a transistor M, resistors, and capacitors as shown in Fig. 5. The supply voltage is V_{DD}. Small-signal input and output voltages are v_{in} and v_{out}, and their Laplace transforms are $V_{\mathrm{in}}(s)$ and $V_{\mathrm{out}}(s)$, respectively. Here, s is a variable of the Laplace transform.

(2) A drain current I_{D} flows through M when the supply voltage V_{DD} is applied. Then, find the maximum value of R_{L} when M operates in the saturation region, by using V_{DD}, V_{TH}, and I_{D}.

(3) When M operates in the saturation region, draw a small-signal equivalent circuit of the circuit in Fig. 5.

(4) When M operates in the saturation region, find a transfer function $\frac{V_{\mathrm{out}}(s)}{V_{\mathrm{in}}(s)}$.

(5) Draw a Bode diagram of the transfer function in Question (4) with respect to the amplitude and the phase. Here, $C_1 R$ is sufficiently larger than $C_2 R_{\mathrm{L}}$.

(6) Find the angular frequency where the amplitude of the transfer function in Question (4) becomes unity at sufficiently high frequency.

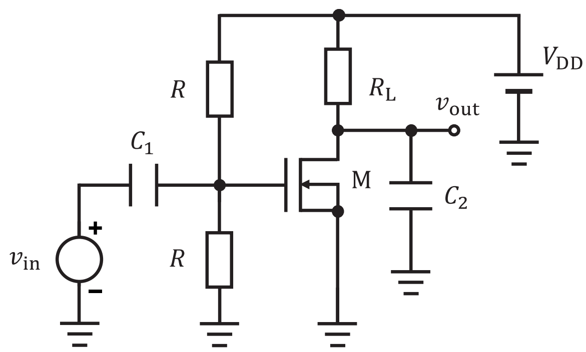
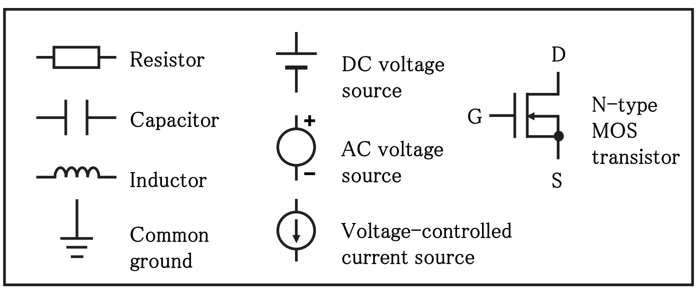


Fig. 5



Legend