## Problem 6

I. Answer the following questions on control systems. Let $t$ denote the time, $s$ denote a variable of the Laplace transform.

Consider the control system as shown in Fig. 1. Let $k$ be a real number, $R(s)$, $U(s)$, and $Y(s)$ denote the Laplace transforms of $r(t)$, $u(t)$, and $y(t)$, respectively.

(1) The plant $G(s)$ is expressed by the following differential equations.

$$u(t) = \frac{d}{dt}x(t) + y(t)$$

$$x(t) = y(t) + \frac{d}{dt}y(t)$$

Let $x(0) = 1$ and $y(0) = 2$, respectively. Express $Y(s)$ using $U(s)$.

(2) Find the final value of $y(t)$ when $u(t)$ is the unit step function.

(3) Find the transfer function $G_1(s) = \frac{Y(s)}{R(s)}$ from the reference $R(s)$ to the output $Y(s)$.

(4) Let reference $r(t)$ be the unit step function. Find the value of $k$ when the output $y(t)$ becomes critically damped.

Next, consider the system as shown in Fig. 2.

(5) Let $G_{\mathrm{C1}}(s) = \frac{K_{\mathrm{I}}}{s}$ and $G_{\mathrm{C2}}(s) = K_{\mathrm{P}} + K_{\mathrm{D}}s$. Find the transfer functions $G_{\mathrm{R}}(s) = \frac{Y(s)}{R(s)}$ from the reference $R(s)$ to the output $Y(s)$, and $G_{\mathrm{D}}(s) = \frac{Y(s)}{D(s)}$ from the disturbance $D(s)$ to the output $Y(s)$.

(6) Let $G_{\mathrm{C1}}(s) = \frac{K_{\mathrm{I}}}{s} + K_{\mathrm{P}} + K_{\mathrm{D}}s$ and $G_{\mathrm{C2}}(s) = 0$. Find the transfer functions $G_{\mathrm{R}}(s) = \frac{Y(s)}{R(s)}$ and $G_{\mathrm{D}}(s) = \frac{Y(s)}{D(s)}$. Also discuss the influences on the $u(t)$, which is the input of the plant $\frac{1}{Ms+D}$, with respect to the changes in the reference $r(t)$, comparing with the result obtained in Question (5).



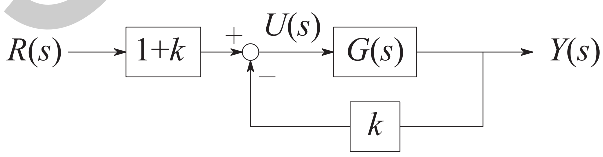


Fig. 1

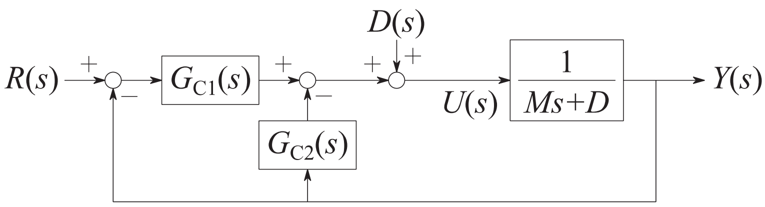


Fig. 2

II. Answer the following questions on an AC circuit. Let $j$ denote the imaginary unit.

Consider the circuit as shown in Fig. 3 which consists of two sinusoidal voltage sources and a reactance. The frequencies of the sinusoidal voltage sources are the same. Let $V_a$ and $V_b$ denote the effective voltage values of nodes A and B, respectively, $X$ denote the reactance, and $\delta$ denote the voltage phase angle difference between nodes A and B.

(1) Find the current flowing from node A to node B.

(2) Find the complex power supplied by the voltage source at node A.

(3) Draw the graph of the active power $P$ supplied by the voltage source at node A as a function of the voltage phase angle difference $\delta$.

(4) Based on the result obtained in Question (3), find the range of $\delta$ so that the active power can be delivered stably. Then explain the reason.

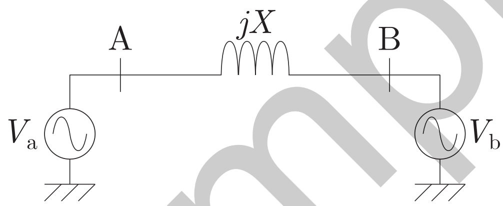


Fig. 3