Problem 2

I.

Answer the following questions on the circuit, with external terminals A and A', shown in Fig. 1. Here, the three resistors in the circuit have the same resistance R. C is a capacitance and L is an inductance. Voltages are represented with respect to the terminal A'.

At first, the switch S is closed. The two voltage sources in the circuit generate the same AC voltage with the amplitude E_0, the angular frequency ω, and the phase angle $\phi = 0$. $\dot{V}_\mathrm{p}$ and $\dot{V}_\mathrm{q}$ are the complex voltage amplitudes (phasors) at the nodes p and q, respectively.

(1) Write the simultaneous equations that $\dot{V}_\mathrm{p}$ and $\dot{V}_\mathrm{q}$ satisfy.

(2) Assuming $R = 1\ \Omega$, $L = 1\ \mathrm{H}$, $C = 1\ \mathrm{F}$, $E_0 = 1\ \mathrm{V}$, and $\omega = 1\ \mathrm{rad/s}$, find $\dot{V}_\mathrm{p}$ and $\dot{V}_\mathrm{q}$.

(3) Under the conditions used in Question (2), an equivalent circuit of Fig. 1 can be represented by an AC voltage source with the complex voltage amplitude $\dot{E}_1$ and an impedance $\dot{Z}$, as shown in Fig. 2. Find $\dot{E}_1$ and $\dot{Z}$.

Now the two voltage sources in Fig. 1 generate the same DC voltage of $1\ \mathrm{V}$. Assume the circuit parameters of $L = 1\ \mathrm{H}$ and $R = 1\ \Omega$. The switch S has been open for a sufficiently long time and then the switch is closed. V_p and V_q are the voltages at the nodes p and q, respectively.

(4) Write the simultaneous differential equations that V_p and V_q satisfy after closing the switch.

(5) After closing the switch, voltages in the circuit exhibit transient variations. Find the range of C for making a damped oscillation occur in the voltage at the node p.

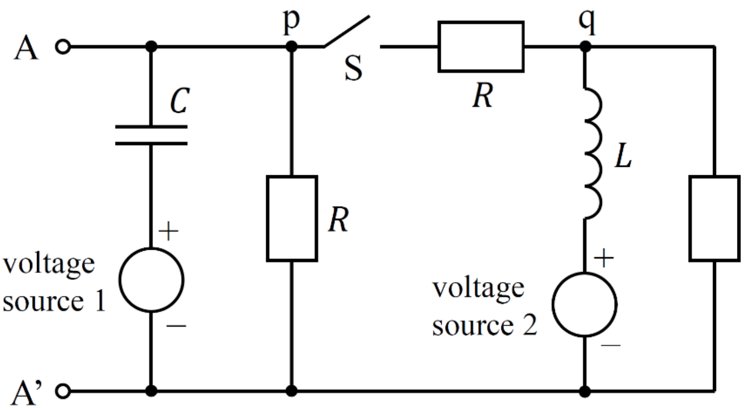


Fig. 1

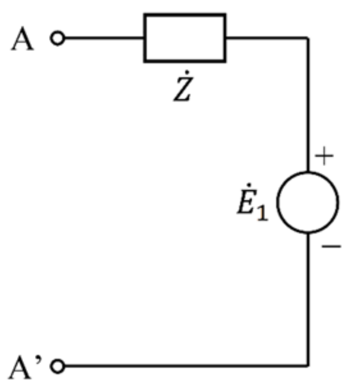


Fig. 2

II.

Answer the following questions on the circuit shown in Fig. 3. The n-type MOS transistors M_1 and M_2 have the same characteristics, and the small-signal equivalent circuit of each transistor is shown in Fig. 4. Here, g_{m}, r_{o}, and v are the transconductance, the drain resistance, and the input voltage of the MOS transistor, respectively. R_{D} and R_{S} are the resistances in the circuit. V_{DD} is a positive constant voltage. Note that v_1, v_2, v_3, and v_4 represent small-signal components of the voltages V_1, V_2, V_3, and V_4, respectively.

(1) The small-signal components v_{in} and v_{out} represent the difference of two input signals $v_1 - v_2$ and the difference of two output signals $v_3 - v_4$, respectively. Find the voltage amplification factor $A = v_{\mathrm{out}}/v_{\mathrm{in}}$.

(2) The small-signal components v'_{in} and v'_{out} represent the sum of two input signals $v_1 + v_2$ and the sum of two output signals $v_3 + v_4$, respectively. Find the voltage amplification factor $A' = v'_{\mathrm{out}}/v'_{\mathrm{in}}$.

(3) Describe in a few lines the function of the circuit in Fig. 3, based on the results of Questions (1) and (2). Assume $r_{\mathrm{o}} \to \infty$ and $R_{\mathrm{S}} \gg g_{\mathrm{m}}^{-1}$.

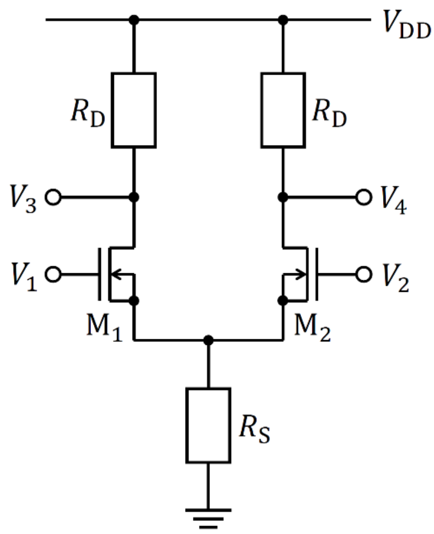


Fig. 3

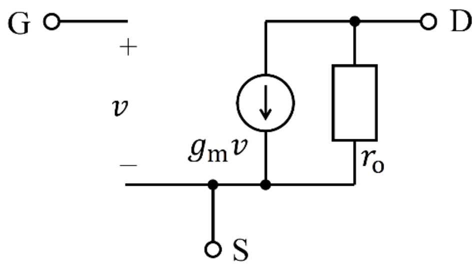
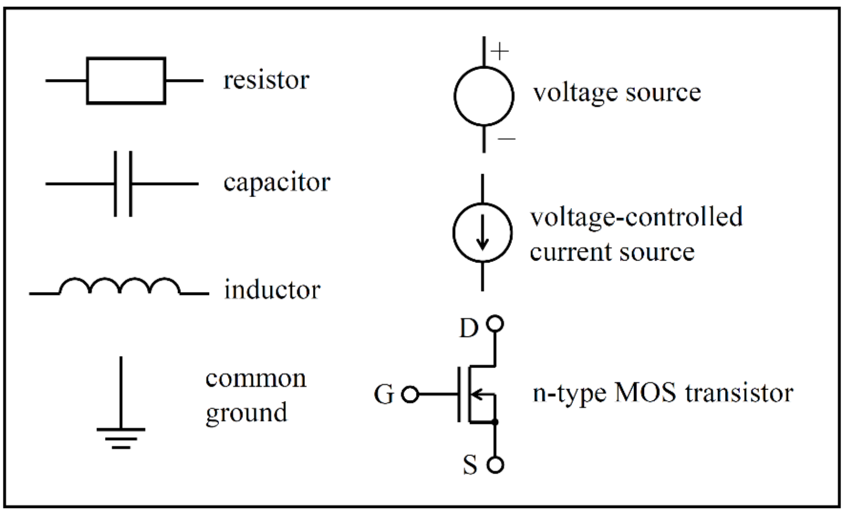


Fig. 4



Legend