Problem 5

I.

Consider an electron in a solid, for which the Schrödinger equation can be given by

$$i\hbar \frac{\partial}{\partial t}\Psi = H\Psi, \qquad \text{(i)}$$

where t is the time, Ψ is the wave function of the electron, H is the Hamiltonian for the electron, i is the imaginary unit, and $\hbar$ is the reduced Planck constant (given by the Planck constant divided by 2π). Answer the following questions. Here, $\omega = E/\hbar$, E is the energy of the electron, and k is the wavenumber of the electron (real number).

(1) Suppose $H = -A\frac{\partial^2}{\partial x^2}$, where A is a real positive constant, and x is the position. Assume that the wave function is given by $\Psi = Ce^{i(kx-\omega t)}$, where C is a constant.

(1-i) Derive the dispersion relationship between E and k.

(1-ii) An electron in this solid can be expressed as a free electron with an effective mass m. Derive m. Note that the momentum of an electron can be given by $\hbar k$.

(1-iii) Consider a one-dimensional solid with a length L. Find the possible values of k in this solid. Here, assume a periodic boundary condition with a period L.

(1-iv) Using the result of Question (1-iii), derive the density of states of an electron at an energy E in this one-dimensional solid.

(2) Suppose $H = -A\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)$, where A is a real positive constant, and x, y, and z are positions. Assume that the wave function is given by $\Psi = Ce^{i(k_x x + k_y y + k_z z - \omega t)}$, where C is a constant, and k_x, k_y, and k_z are the wavenumber components in the x-, y-, and z-direction, respectively. Derive the density of states of an electron at an energy E in this three-dimensional solid.

(3) Suppose $H = \begin{pmatrix} 0 & Bk \\ Bk & 0 \end{pmatrix}$, where B is a positive real number. In this case, the Schrödinger equation for an electron can be given by

$$i\hbar \frac{\partial}{\partial t}|\Psi\rangle = H|\Psi\rangle, \qquad \text{(ii)}$$

where $|\Psi\rangle$ is the state vector of the electron.

(3-i) Suppose $|\Psi\rangle = \begin{pmatrix} \varphi_1 \\ \varphi_2 \end{pmatrix} e^{-i\omega t}$, where φ_1 and φ_2 are time-independent values. Derive the dispersion relationship between E and k.

(3-ii) Derive the propagation speed of this electron given by dE/dk, and discuss the difference between this electron and a free electron.

II.

Consider a silicon (Si) p-n junction biased with a voltage V as shown in Fig. 1. An n-type Si and a p-type Si are uniformly doped with phosphorus (P) and boron (B), respectively. The doping concentrations of the n-type Si and the p-type Si are N_D and N_A, respectively. The activation rate of the dopants is 100%. Here, x is the position, and the interface between the n-type Si and the p-type Si is located at $x = 0$. The edges of the depletion layer are at $x = -l_n$ and $x = l_p$. Here, k_B is the Boltzmann constant, q is the elementary charge, T is the temperature of the p-n junction, ε_s is the permittivity of Si, and n_i is the intrinsic carrier density of Si. Answer the following questions, assuming that T is at room temperature.

(1) Draw the band diagram of the p-n junction when $V = 0$. The Fermi level E_F and the intrinsic Fermi level E_i must be indicated in the band diagram.

(2) Derive the built-in potential of the p-n junction V_{bi}. Here, the electron density at the thermal equilibrium condition can be given by $n_i e^{(E_F - E_i)/k_B T}$, and the hole density by $n_i e^{(E_i - E_F)/k_B T}$.

(3) Derive and sketch the electro-static potential and electric field in the depletion layer using l_n and l_p when $V = 0$. Here, the electric field can be assumed to be zero in the n-type and p-type neutral regions.

(4) Derive the depletion layer width $W = l_n + l_p$ using V_{bi} when $V = 0$.

(5) Draw the band diagram under a forward bias ($V > 0$), and explain the reason why a current increases exponentially with respect to V using the energy distribution of electrons in the conduction band.

(6) Under a reverse bias ($V \ll 0$), there is a voltage range where a constant, non-zero current flows. Explain this reason using the distributions of minority carriers in the neutral regions.

(7) Suppose crystal defects uniformly distribute in Si. Such crystal defects can be generation centers or recombination centers of electrons and holes. The effective recombination rate at the crystal defects per a unit volume is given by $\frac{np - n_i^2}{n + p + 2n_i} \cdot \frac{1}{\tau}$, where n is the electron density, p is the hole density, and τ is the carrier lifetime. Derive the generation current density using the depletion layer width W under a reverse bias.

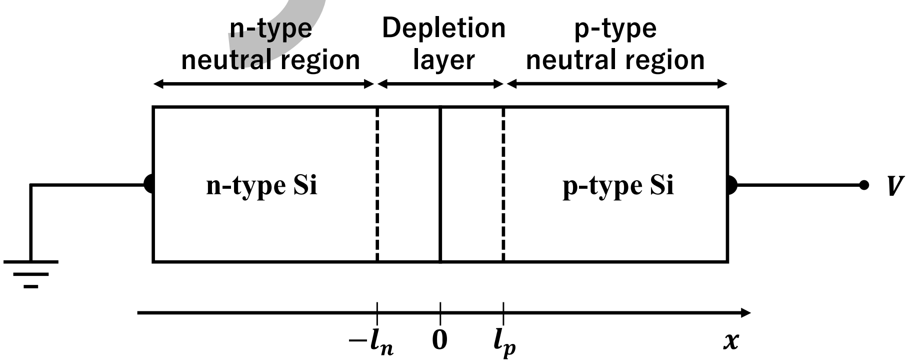

Fig. 1