Problem 6

I.

Consider a dynamic plant expressed by the following differential equation:

$$\frac{d^2y(t)}{dt^2} + 3\frac{dy(t)}{dt} = u(t) - 2y(t) \ ,$$

where $u(t)$ and $y(t)$ are the input and the output of the control system, respectively, and the corresponding Laplace transforms are denoted by $U(s)$ and $Y(s)$. Answer the following questions.

(1) Find the transfer function of this plant $G_0(s) = \frac{Y(s)}{U(s)}$.

(2) Find the time response of $y(t)$ when the input $u(t)$ is a unit step function. Then find the final value of the output response.

Next, consider the feedback control system shown in Fig. 1. The transfer function of the controller is given by $G_\mathrm{c}(s) = 1$, and the transfer function of the plant is given by $G_\mathrm{p}(s) = \frac{K}{s(Ts+1)}$, where $K > 0$, $T > 0$. Here $R(s)$ and $C(s)$ denote the reference input and the controlled variable, respectively, which are expressed by the Laplace transformation of time signals $r(t)$ and $c(t)$.

(3) Find the transfer function $G_1(s)$ of the whole control system in Fig. 1.

(4) Find the natural angular frequency ω_n and the damping ratio ζ of the transfer function $G_1(s)$ obtained in Question (3).

(5) Explain how the behavior of the system response to a unit step input changes as T is increased in the feedback control system in Fig. 1.

(6) <u>Figure 2 shows the control system where an inner feedback loop is added to the system in Fig. 1.</u> Here, $H(s) = K_\mathrm{c}\ s$, where $K_\mathrm{c} > 0$. Find the transfer function $G_2(s)$ of the whole control system in Fig. 2.

(7) Show that the stability of the system improves when K_c is sufficiently large in the feedback control system obtained in Question (6).

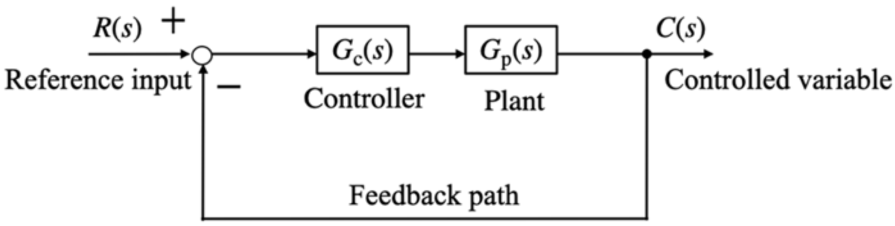

Fig. 1

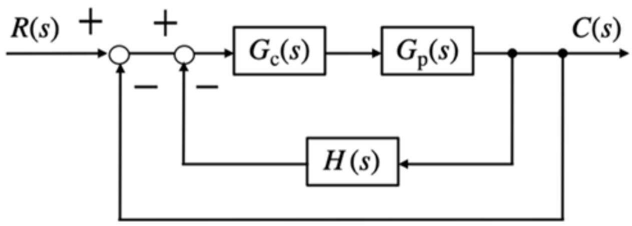

Fig. 2

II.

Consider an equivalent circuit of the DC shunt motor shown in Fig. 3 with a constant supply voltage V. Here, r_a is the armature resistance including the compensating winding resistance, r_f is the field resistance, v_b is the brush voltage drop, I is the input current, I_a is the armature current, I_f is the field current, and E is the induced electromotive force. We assume $V = 220$ [V], $r_a = 0.1$ [Ω], $r_f = 110$ [Ω], and $v_b = 3$ [V]. The magnetic saturation and the armature reaction are assumed to be negligible. Answer the following questions.

(1) Find I_a and E when $I = 72$ [A].

(2) Find the torque T_M of the motor, when $I = 72$ [A] and the rotation speed of the motor is 1,200 rpm.

(3) Assume a case where r_f is doubled while keeping T_M obtained in Question (2). Show that I_a doubles.

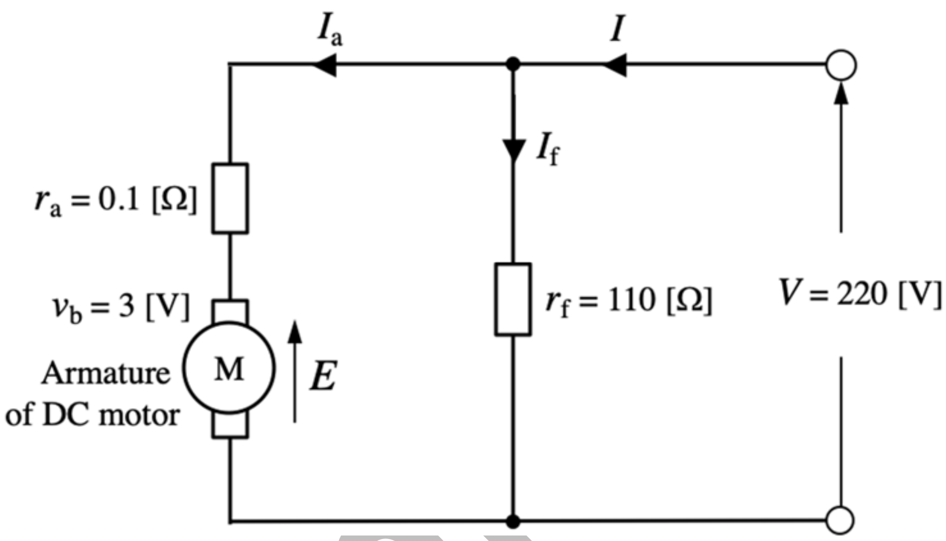

Fig. 3