Problem 1

I. Answer the following questions. In answers, show the process for derivation as well. The permittivity of the space is the permittivity of the vacuum, ε_0.

As shown in Fig. 1, the space between hollow coaxial cylindrical conductors with radii a and b $(a > b)$ is filled with a uniform dielectric material with a permittivity of $8\varepsilon_0$. Here, the outer cylindrical conductor (radius a) is grounded, and the voltage applied to the inner cylindrical conductor (radius b) is V $(V > 0)$. The coaxial cylindrical conductors are sufficiently long, and the edge effect can be ignored.

(1) Consider the case where electric charges per unit length of coaxial cylindrical conductors are $+q$ and $-q$ $(q > 0)$. Express the amplitude of electric flux density D in the dielectric at a distance r from the central axis O using q and r.

(2) Express the electric field intensity E in the dielectric using V, r, a, and b. Draw a graph with E on the vertical axis and r on the horizontal axis.

(3) Express the electrostatic capacitance per unit length using a, b, and ε_0.

(4) Let $E_{\max}$ be the maximum value of the electric field intensity E. When the applied voltage V and the radius a of the outer cylindrical conductor are fixed, find the radius b of the inner cylindrical conductor that gives the minimum value of $E_{\max}$, and write the expression of $E_{\max}$ in that case.

(5) In Question (4), replace the dielectric between the coaxial cylindrical conductors with a dielectric with a permittivity $\varepsilon_1(r)$, which is a function of r, and $\varepsilon_1(b) = 5\varepsilon_0$. Find the permittivity $\varepsilon_1(r)$ that minimizes $E_{\max}$, and write the expression of $E_{\max}$ in that case.

Next, consider the case where a small spherical void exists in the dielectric with a permittivity of ε_2 (constant, $\varepsilon_2 > \varepsilon_0$), as shown in Fig. 2. Here, a void is an empty space. Assume that the permittivity of the void is ε_0.

(6) Draw sketches of the electric flux lines (flux lines of electric flux density $\boldsymbol{D}$) and the electric lines of force (flux lines of electric field $\boldsymbol{E}$) in and around the void in Fig. 2.

(7) Answer whether the amplitude of electric field in the void becomes larger or smaller than that without a void. Explain the reason briefly in about two lines of text.

Sample

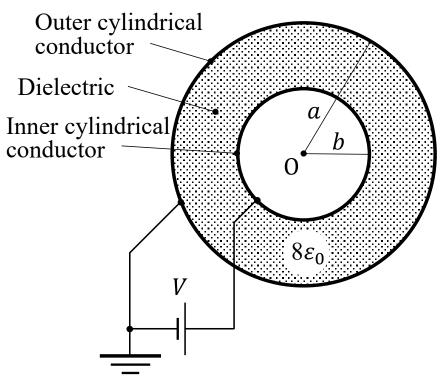


Fig. 1

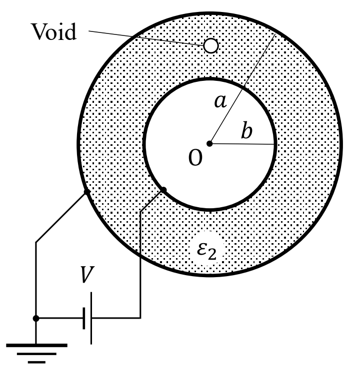


Fig. 2

II. Answer the following questions. In answers, show the process for derivation as well. The permeability of the space is the permeability of the vacuum, μ_0.

Consider the magnetic field generated by a straight line current in a Cartesian coordinate system (x, y, z). As shown in Fig. 3, the line current I flows in the positive direction along the z-axis.

(1) Find the magnetic flux density $\boldsymbol{B} = (B_x, B_y, B_z)$ at a point (x, y, z).

(2) Show that one of the magnetic vector potentials giving the magnetic flux density $\boldsymbol{B}$ is $\boldsymbol{A} = (0, 0, A_z)$.

Next, as shown in Fig. 4(a), a current I flows in a straight conductor at a height h from the earth. The straight conductor is set in parallel to the earth, and its current density is uniform in the circular cross-section of diameter $2a$. Here, a is sufficiently smaller than h $(a \ll h)$. The x- and z-axes are on the earth's surface, and the $+z$ direction coincides with the direction of current flow. The center of the cross-section of the conductor through which the current flows is on the line $x = 0$, $y = h$. The earth can be considered a perfect conductor. As shown in Fig. 4(b), when the earth is replaced with an image current $-I$ placed at a distance h below the earth's surface, magnetic flux density $\boldsymbol{B}$ in the region $y > 0$ is equal to the sum of the magnetic flux density $\boldsymbol{B}_1$ generated by the current I and the magnetic flux density $\boldsymbol{B}_2$ generated by the image current $-I$, namely, $\boldsymbol{B} = \boldsymbol{B}_1 + \boldsymbol{B}_2$.

(3) The external magnetic flux Φ_{o} is defined as the magnetic flux passing between the surface of the straight conductor and the earth's surface. When considering a unit length in the z direction, it is expressed by the following equation, which is a surface integral in the plane of $x = 0$.

$$\Phi_{\mathrm{o}} = \int_0^1 \int_0^{h-a} \boldsymbol{B} \cdot \boldsymbol{n} \, dy \, dz$$

Here, $\boldsymbol{n}$ is the unit normal vector to the area element $dydz$ for the surface integral. Find the external magnetic flux Φ_{o} and the external inductance per unit length of the straight conductor L_{o}.

(4) The inductance of this straight conductor is the sum of the external inductance L_{o} and the internal inductance L_{i}, which is related to the internal magnetic flux. Express the magnetic energy of this system using L_{o}, L_{i}, and I.

(5) In Question (4), assume that the internal inductance L_{i} is a constant. Using the magnetic energy and virtual displacement, find the direction and magnitude of electromagnetic force per unit length acting on the straight conductor.

Sample

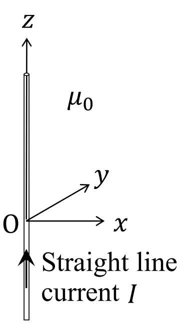
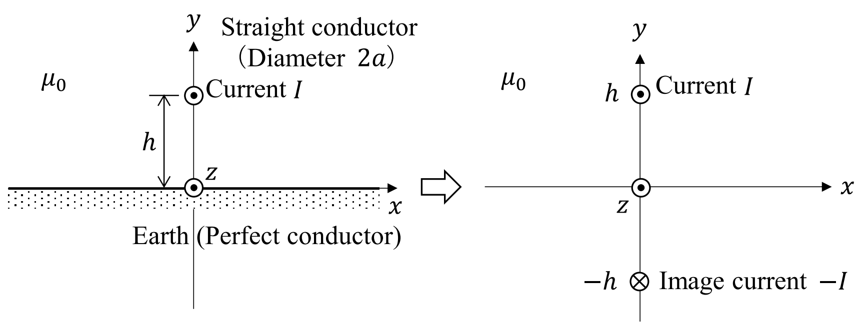


Fig. 3



(a) (b)

Fig. 4