## Problem 2

I. Answer the following questions on electric circuits. Here, $s$ is a variable of the Laplace transform. Assume that $L_1 > M$ and $L_2 > M$ hold among the self inductances $L_1$ and $L_2$, and the mutual inductance $M$. Symbols in the figures are defined in the legend.

(1) As shown in Fig. 1, the currents $I_1(s)$ and $I_2(s)$ flow in two inductors with the self inductances $L_1$ and $L_2$, respectively, and with the mutual inductance $M$. Find the self inductances $L_\alpha$, $L_\beta$, and $L_\gamma$ of the circuit in Fig. 2 that is equivalent to that in Fig. 1, using $L_1$, $L_2$, and $M$.

(2) In the AC bridge circuit shown in Fig. 3 with the angular frequency $\omega$, no current flows through the AC detector D. Find the self inductance $L_2$ and the mutual inductance $M$. You may use the resistances $R_\mathrm{A}$, $R_\mathrm{B}$, $R_\mathrm{C}$, the self inductance $L_1$, the capacitance $C$, and the angular frequency $\omega$, if necessary.

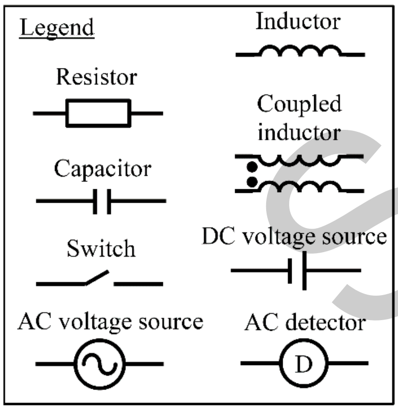
The circuit in Fig. 4 contains the DC voltage source $E$, the two identical resistors $R$, and the capacitor $C$. Assume that $L_1 = L_2 = \sqrt{2}M$ holds in the following questions. The switch S had been opened for a sufficiently long time and then was closed at time $t = 0$. Dumped oscillations are observed at $t > 0$ in the currents flowing in the circuit.

(3) Write the simultaneous equations that the currents $I_1(s)$, $I_2(s)$, and $I_3(s)$ satisfy at $t > 0$.

(4) Find $I_3(s)$. It is not required to transform it to a time-domain form.

(5) Find the range of capacitance $C$ for generating the damped oscillation.





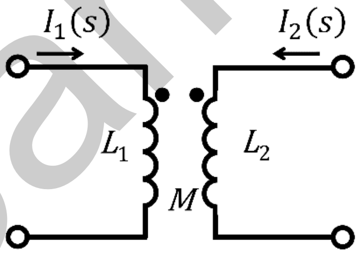




Fig. 1

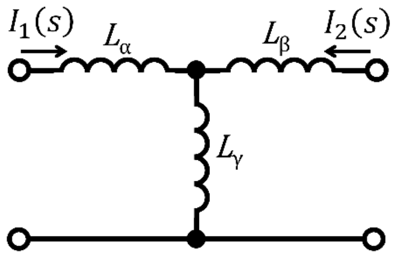



Fig. 2

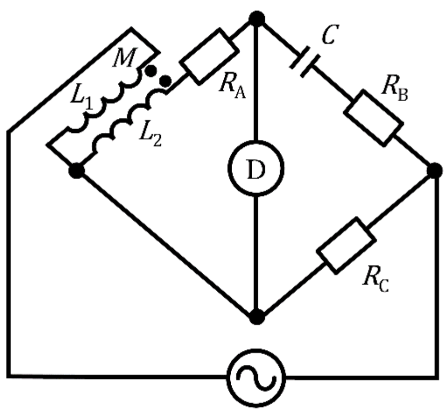



Fig. 3

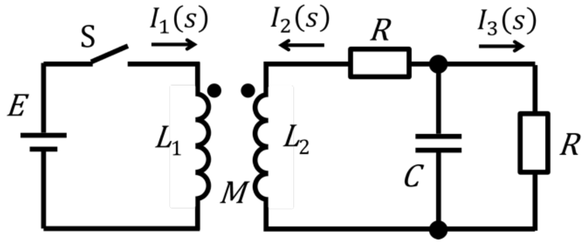



Fig. 4

II. Answer the following questions on circuits using N-type MOS transistors. The symbols in the figures are defined in the legend. Use the circuit in Fig. 5 as a small-signal equivalent circuit for an N-type MOS transistor. Here, $g_\mathrm{m}$ and $v_\mathrm{GS}$ are the transconductance and the gate-to-source voltage, respectively.

(1) Draw a small-signal equivalent circuit for the circuit in Fig. 6.

(2) The circuit in Fig. 6 receives a small-signal AC input $v_\mathrm{in}$, which is a complex voltage with angular frequency $\omega$. Find the complex output voltage $v_\mathrm{out}$.

(3) Choose out of the following A and B a correct description regarding the characteristics of the circuit in Fig. 6. Briefly explain this reason. In addition, find the cutoff angular frequency of the circuit.

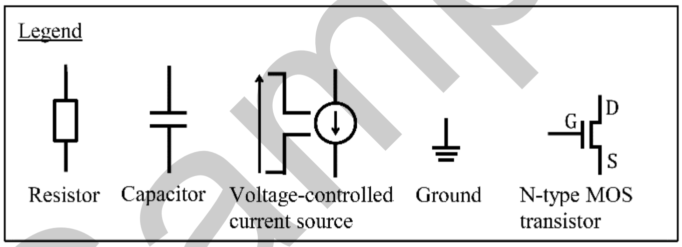
A. The circuit works as a low-pass filter.

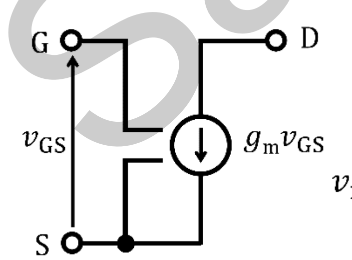
B. The circuit works as a high-pass filter.

The circuit in Fig. 7 consists of the three circuits in Fig. 6. Oscillatory voltages occurred in this circuit. Here, the voltages in this circuit can be regarded as small signals.

(4) Find the angular frequency of the oscillation.

(5) Write the relation that the resistance $R_\mathrm{L}$ and the transconductance $g_\mathrm{m}$ satisfy for generating the oscillatory voltage.



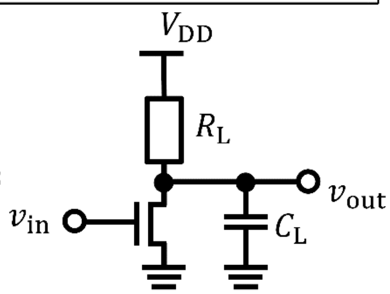




Fig. 5 Fig. 6

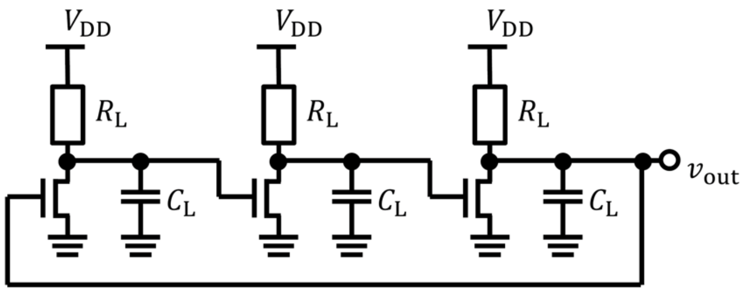



Fig. 7