Problem 5

I. Consider a particle with a kinetic energy E injected from $x = -\infty$ to a potential barrier as shown in Fig. 1. The potential barrier is given by

Region I : $V(x) = 0$ $\quad x < 0$

Region II : $V(x) = V_0\ (> 0)$ $\quad 0 \leqq x \leqq w$

Region III : $V(x) = 0$ $\quad x > w.$

We consider the reflection and transmission probability of the particle due to the potential barrier.

(1) Let us consider the case of $E > V_0$.

(1-i) The time-independent Schrödinger equation for the particle's wave function $\Phi(x)$ is given by

$$-\frac{\hbar^2}{2m}\frac{d^2\Phi(x)}{dx^2} + V(x)\Phi(x) = E\Phi(x). \qquad \text{(i)}$$

Here, m is the particle's mass, $\hbar$ is the Planck constant divided by 2π. $\Phi(x)$ in the regions I, II, and III is given by

Region I : $\Phi(x) = Ae^{ikx} + Be^{-ikx}$

Region II : $\Phi(x) = Ce^{i\alpha x} + De^{-i\alpha x}$

Region III : $\Phi(x) = Fe^{ikx}$.

Here, i is the imaginary unit, k and α are positive real numbers. A, B, C, D, and F are complex constants. Express k and α using necessary parameters from among $\hbar, m, E,$ and V_0.

(1-ii) Write the equations for the boundary conditions of $\Phi(x)$ at $x = 0$ and $x = w$. From these equations, express A, B, C, and D in the forms that are proportional to F.

(1-iii) Show that the reflection probability R and transmission probability T of the incident particle injected from $x = -\infty$ are given by the following equations (ii) and (iii), respectively.

$$R = \left[1 + \frac{4E(E - V_0)}{{V_0}^2 \sin^2(\alpha w)}\right]^{-1} \qquad \text{(ii)}$$

$$T = \left[1 + \frac{{V_0}^2 \sin^2(\alpha w)}{4E(E - V_0)}\right]^{-1} \qquad \text{(iii)}$$

(1-iv) The transmission probability T is not always 1 even in the case of $E > V_0$. Find the condition for $T = 1$. Explain the reason why it is so from the viewpoint of the difference between classical mechanics and quantum mechanics, in about five lines.

(2) Let us consider the case of $E < V_0$. Find the transmission probability T of the incident particle. T is not zero even when $E < V_0$. Explain the reason why it is so from the viewpoint of the difference between classical mechanics and quantum mechanics, in about five lines.

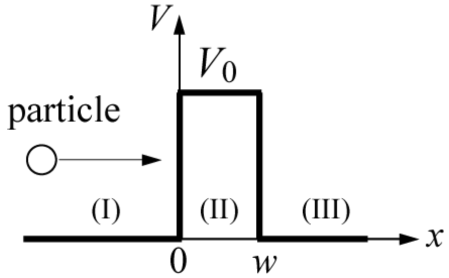

Fig. 1

II. Fig. 2(a) shows the energy band diagram of a metal and an N-type semiconductor before forming a contact. In the N-type semiconductor, the donor density is N_D and the activation rate is assumed to be 100% (That is, all the donors are ionized and each donor supplies one electron). The vacuum levels E_S in both sides are at the same energy. E_{FM} is the Fermi level of the metal. E_C, E_V, and E_{FS} are the conduction band bottom, the valence band top, and the Fermi level of the N-type semiconductor, respectively. $q\Phi_M$ and $q\Phi_S$ are the work functions of the metal and semiconductor, respectively. $q\chi$ is the electron affinity. Assume that $q\Phi_M$, $q\Phi_S$, and $q\chi$ are constant values determined by the materials. q is the elementary charge.

(1) Consider the case $\Phi_M > \Phi_S$ (namely, a Schottky contact). Draw the band diagram, including E_S, E_C, E_V, E_{FS}, and E_{FM} when the metal and the semiconductor are brought into contact. Here, we assume an ideal interface without any surface oxide layer or dangling bonds.

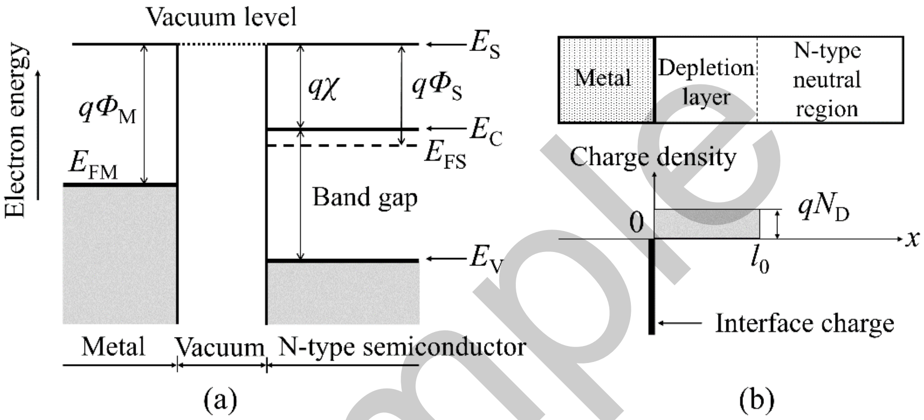

Fig. 2

Consider charge distribution at the Schottky contact given in Question (1). A depletion layer is normally formed in the N-type semiconductor up to a distance l_0 from the interface, as shown in Fig. 2(b). In this depletion layer ($0 < x < l_0$), there are positive space charges from the ionized donors that are uniformly distributed with the density N_D. The region where $x > l_0$ is called N-type neutral region. Meanwhile, there are negative charges with a surface density of $-Q$ distributed at the metal side of the interface.

(2) Using the parameters given in Fig. 2(a), express the potential barrier heights at the metal/semiconductor interface for the electron carriers in the metal and in the N-type neutral region. We assume that the electron carriers supplied by the donors in the semiconductor are at the conduction band bottom.

(3) Find the depletion layer thickness l_0 and the magnitude of surface charge density Q accumulated at the metal side of the interface. If necessary, use the vacuum permittivity ε_0 and the relative permittivity ε_r of the N-type semiconductor.

(4) We connect the positive and negative electrodes of a DC voltage source to the metal side and the semiconductor side, respectively. When we apply a bias voltage V and measure the current I, a rectification effect appears. Draw the shape of the $I - V$ characteristics in this case and explain the reason for this $I - V$ characteristics.

(5) In device operations, the high contact resistance due to the abovementioned rectification effect usually causes problems. Describe method(s) to decrease this high contact resistance.