Problem 6

I. Answer the following questions regarding control systems. Let t denote the time and s denote a variable of the Laplace transform. The Laplace transform of a function $f(t)$ is defined as the following equation.

$$F(s) \equiv \int_0^\infty f(t)e^{-st}\,dt$$

(1) Find the Laplace transforms of the functions below by following the above definition. Show the processes for derivation.

(1-i) $f_1(t) = 0\ (t < 0),\ f_1(t) = 1\ (t \geq 0)$

(1-ii) $f_2(t) = 0\ (t < 0),\ f_2(t) = e^{-\alpha t}\cos(\beta t)\ (t \geq 0,\ \alpha$ and β are positive real numbers.)

(1-iii) $f_3(t) = 0\ (t < 0),\ f_3(t) = t^n\ (t \geq 0,\ n$ is a natural number.)

(2) Find the inverse Laplace transforms of the functions below.

(2-i) $F_4(s) = \frac{1}{s^3}$

(2-ii) $F_5(s) = \frac{1}{s^2+2s+3}$

(2-iii) $F_6(s) = \frac{1}{(s+2)^2(s+1)}$

(3) Let us consider a system expressed by the following differential equation. Find $Y(s)$ using $U(s)$ in the case of $y(0) = 0$. $Y(s)$ and $U(s)$ denote the Laplace transforms of the output $y(t)$ and the input $u(t)$, respectively.

$$u(t) = \frac{d}{dt}y(t) + y(t)$$

(4) Let $G_2(s) = \frac{Y(s)}{U(s)}$, from the result obtained in Question (3). Answer the following questions regarding the system shown in Fig. 1. $R(s)$ and $D(s)$ denote the Laplace transforms of the reference $r(t)$ and the disturbance $d(t)$, respectively.

(4-i) Let $G_1(s) = K_1$ in Fig. 1. K_1 is assumed to be a positive real number here. Find transfer functions from $R(s)$ to $Y(s)$ and from $D(s)$ to $Y(s)$.

(4-ii) Find the steady state error in Question (4-i) when $r(t)$ is the unit step function and $d(t)$ is 0.

(4-iii) Let $G_1(s) = K_1 + \frac{K_2}{s}$ in Fig. 1. K_1 and K_2 are assumed to be positive real numbers. Find the steady state error when $r(t)$ is the unit step function and $d(t)$ is 0.

(4-iv) Let $K_2 = 1$ in Question (4-iii). Find the conditions on K_1 when the unit step responses are damped oscillation, critical damping, and overdamping. Draw the root locus for the transfer function from $R(s)$ to $Y(s)$ when K_1 becomes larger from a very small positive value, and briefly explain the relationships between the location of the poles and the step response.

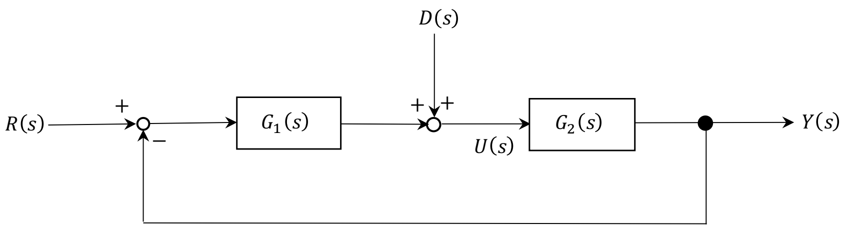

Fig. 1

II. Answer the following questions regarding AC circuits. In the following questions, transmission lines are assumed to be included in the loads.

(1) Express the whole active power P transmitted in the symmetrical three-phase three-wire system in Fig. 2, using the line voltage V, the line current I, and the power factor of the load $\cos\varphi$. Note that V and I are effective values, which are real numbers.

(2) In Question (1), let W be the thermal loss of one transmission line, and $\frac{3W}{P}$ be the rate of the thermal loss. Express transmission efficiency, $\mu = 1 - \frac{3W}{P}$, using the resistance of one transmission line, $R_{3\mathrm{p}}$, as well as P, V, and $\cos\varphi$.

(3) Consider two cases of transmitting the same active power P using the symmetrical three-phase three-wire system in Fig. 2 and the single-phase two-wire system in Fig. 3. Assume that both the transmission systems have the same values of the line voltage V, the resistivity of the transmission lines ρ, the length of each transmission line l, the power factor of the load $\cos\varphi$, and the thermal loss of one transmission line W. The resistance of one transmission line in Fig. 3 is R_{sp}. Express the necessary total cross-sectional area S of the transmission lines in both the power transmission systems, using V, P, ρ, l, $\cos\varphi$, and W. From the above calculated results, briefly explain implications regarding the comparison.

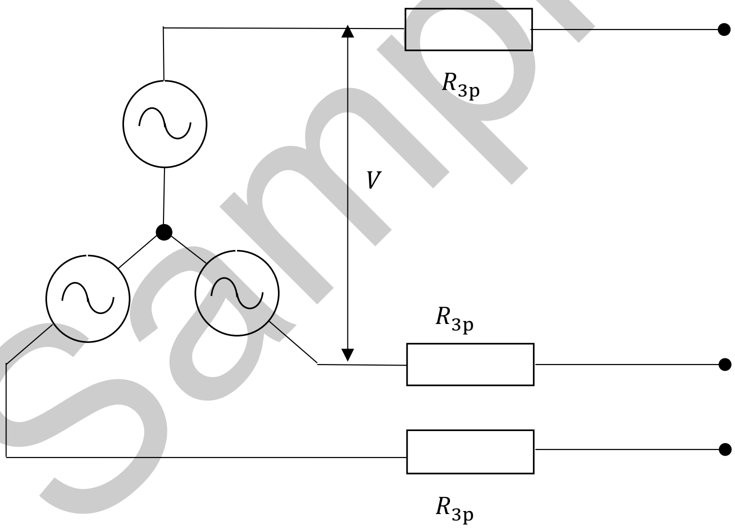

Fig. 2

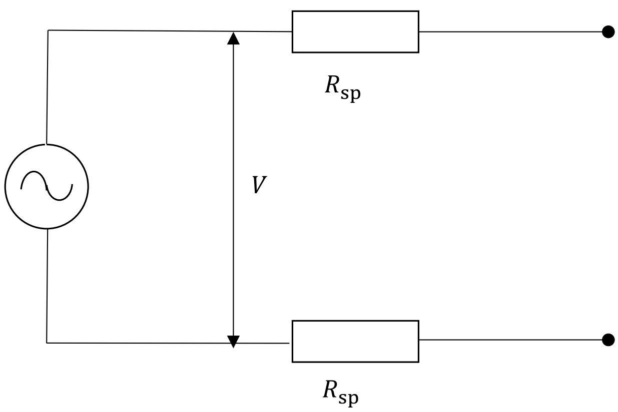

Fig. 3