Problem 1

I.

Let us consider a stacked parallel-plate structure composed of two dielectrics and three metal electrodes, as shown in Fig. 1. Let us call the bottom and top dielectrics dielectric 1 and 2, respectively. Dielectrics 1 and 2 have dielectric permittivities ε_1 and ε_2 and thicknesses d_1 and d_2, respectively. A DC voltage source and a pulsed voltage source are connected to the top and bottom electrodes, as shown in Fig. 1. In the initial condition, the intermediate electrode sandwiched by dielectric 1 and 2 is electrically neutral. Suppose that dielectric 1 has such a property that charges can flow through it when a voltage larger than a certain value is applied between the intermediate and the bottom electrodes, but that such charge flow does not occur in dielectric 2. We can neglect the edge effect of the structure. Answer the following questions.

(1) When the pulsed voltage source is shorted and only a DC voltage V is applied between the top and the bottom electrodes, no charge flow is observed in dielectric 1. Obtain the sheet charge densities (charge per unit area) accumulated on the top electrode, q_{t0}, and on the bottom electrode, q_{b0}.

(2) Next, in addition to the DC voltage, we apply a single rectangular pulsed voltage of a duration Δt, whose polarity is the same as that of the DC voltage. When the pulsed voltage is applied, charge transfer takes place from the intermediate electrode to the bottom electrode through dielectric 1 and we observe a constant current density J during Δt. After the end of the pulsed voltage, the charge state of the intermediate electrode is preserved and the sheet charge density accumulated on the bottom electrode becomes q_{b}. Calculate q_{b}.

(3) As considered in Question (2), after the charge transfer takes place from the intermediate electrode to the bottom electrode, the sheet charge density accumulated on the bottom electrode changes from q_{b0} to q_{b}. To make q_{b} equal to q_{b0}, we need to adjust the DC voltage from V to $V + \Delta V$. You may assume that there is no current flow through dielectric 1 at $V + \Delta V$. Calculate ΔV.

(4) Discuss for what kind of device applications you can use the effect described here.

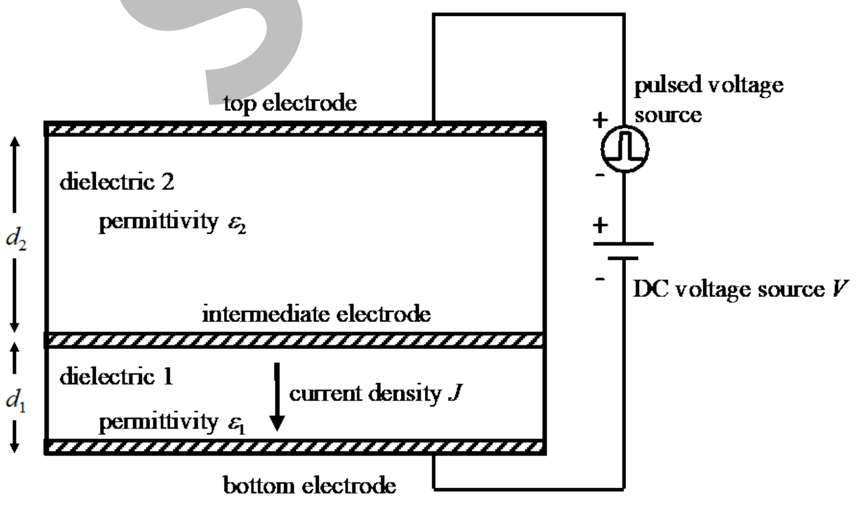

Fig. 1

II.

Let us take the xyz-coordinates as shown in Fig. 2 and consider a system in which two semi-infinitely large dielectrics 1 and 2 are touching with each other in the xy-plane. Dielectrics 1 and 2 have dielectric permittivities ε_1 and ε_2, respectively ($\varepsilon_2 > \varepsilon_1$). The two dielectrics have magnetic permeability μ_0. Consider a case in which an electromagnetic plane wave with an electric field component $\boldsymbol{E}_{\mathbf{i}} = (E_{\mathrm{i}}, 0, 0)e^{i(k_{\mathrm{i}}z-\omega t)}$ is propagating in dielectric 1 in the $+z$-direction and incident to the interface between the two dielectrics. Here, k_{i} is the wave number of the incident electromagnetic wave, ω is the angular frequency, z is the position coordinate on the z-axis, t is time, and i is the imaginary unit. The electric and magnetic fields of the electromagnetic waves have only components which are parallel to the xy-plane and they are uniform in the in-plane direction. Also, we can neglect losses and assume that there is no free current in the dielectrics or at the interface. Answer the following questions.

(1) By using Maxwell's equations, show that the magnetic field component of the incident electromagnetic wave, $\boldsymbol{H}_{\mathbf{i}}$, that propagates in dielectric 1 can be expressed as Eqs. (i) and (ii);

$$\boldsymbol{H}_{\mathbf{i}} = (0, H_{\mathrm{i}}, 0)e^{i(k_{\mathrm{i}}z-\omega t)} \qquad \text{(i)}$$

$$H_{\mathrm{i}} = Y_1 E_{\mathrm{i}} \qquad \text{(ii)}$$

Here, $Y_1 \equiv \sqrt{\frac{\varepsilon_1}{\mu_0}}$ and is called the admittance of dielectric 1.

(2) At the interface between dielectric 1 and dielectric 2, a part of the incident electromagnetic wave is reflected and the rest is transmitted. Let us write the electric field of the reflected wave as $\boldsymbol{E}_{\mathbf{r}} = (E_{\mathrm{r}}, 0, 0)e^{-i(k_{\mathrm{r}}z+\omega t)}$ and that of the transmitted wave as $\boldsymbol{E}_{\mathbf{t}} = (E_{\mathrm{t}}, 0, 0)e^{i(k_{\mathrm{t}}z-\omega t)}$. Here, k_{r} and k_{t} are the wave numbers of the reflected and the transmitted electromagnetic waves, respectively. Write down the boundary conditions for the electric and magnetic fields at the interface of dielectrics 1 and 2. Furthermore, express E_{r} and E_{t} by using E_{i}. If necessary, define the admittance of dielectric 2, Y_2, and use it in the calculation.

(3) Calculate the ratio of the reflected power to the incident power (reflection coefficient, R) and the ratio of the transmitted power to the incident power (transmission coefficient, T).

Next, we insert an infinitely thin conductive layer between the two dielectrics, as shown in Fig. 3. Due to this conductive layer, a current per unit width, $\boldsymbol{J} = \sigma\boldsymbol{E}$, flows at the interface and, therefore, the boundary conditions for the electric and magnetic fields at the interface need to be modified. Here, σ is a constant related to the conductivity of the conductive layer and $\boldsymbol{E}$ is the electric field component of the electromagnetic wave at the interface.

(4) Write down the boundary conditions for the electric and magnetic fields that need to be fulfilled at the interface of the two dielectrics when the conductive layer is inserted.

(5) Obtain the reflection coefficient R and the transmission coefficient T. Furthermore, calculate the ratio of the power absorbed in the conductive layer to the incident power (absorption coefficient, A).

(6) Explain how and why A changes when σ is increased from 0 to infinity. Furthermore, calculate the value of σ, when A becomes maximal, and also calculate the maximal A.

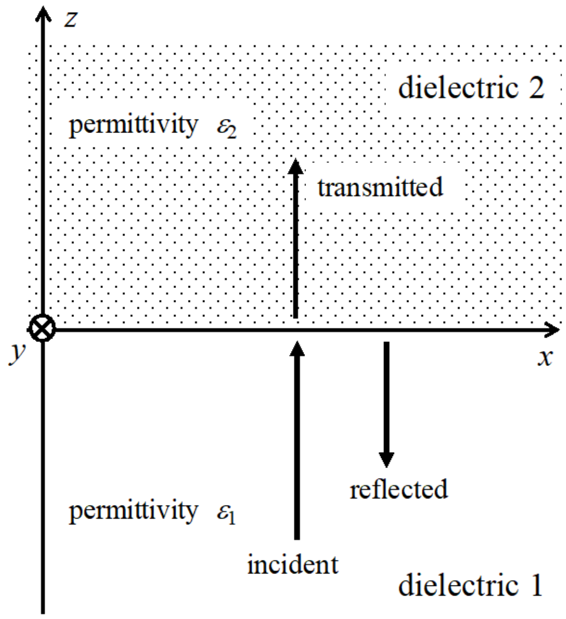
z
dielectric 2
permittivity ε_2
transmitted
y
x
reflected
permittivity ε_1
incident
dielectric 1

Fig. 2

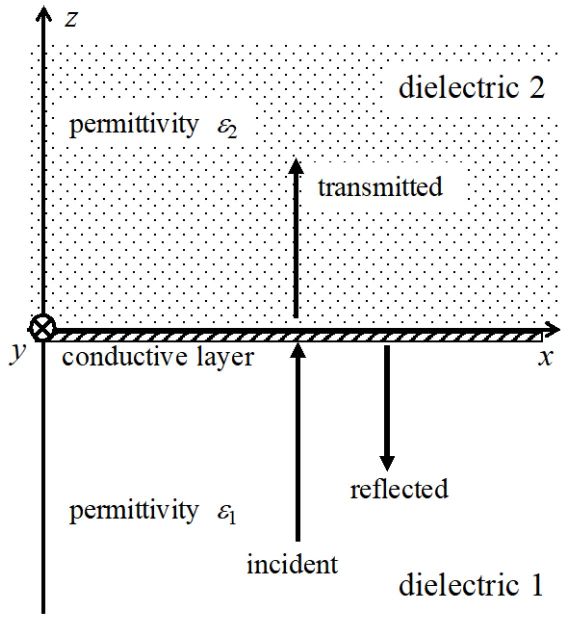
z
dielectric 2
permittivity ε_2
transmitted
y
conductive layer
x
reflected
permittivity ε_1
incident
dielectric 1

Fig. 3