Problem 5

I.

Suppose that the potential in an one-dimensional space along the x-axis is given as $V(x) = 0$ for Region A ($x < 0$) and $V(x) = V_0$ ($V_0 > 0$) for Region B ($x \geq 0$), as shown in Fig. 1. Also suppose a wave function $\Psi(x, t) = \phi(x)e^{-i\omega t}$ for a particle with mass m in this space. Here, i is the imaginary unit, ω is the angular frequency and t is time. $\phi(x)$ is given as a solution of the time-independent Schrödinger equation:

$$\left\{-\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + V(x)\right\}\phi(x) = E\phi(x), \qquad \text{(i)}$$

where E ($E > 0$) is the energy of this particle and $\hbar$ is the reduced Planck constant (given by the Planck constant divided by 2π). Answer the following questions.

(1) Prove that a general solution of Eq. (i) in Region A is given by the following equation (ii) with arbitrary constants C_1 and C_2. In addition, give an expression for α, where α is a positive real number.

$$\phi(x) = C_1 e^{-i\alpha x} + C_2 e^{i\alpha x}. \qquad \text{(ii)}$$

(2) Give a general solution of $\phi(x)$ in Region B satisfying Eq. (i) when $E < V_0$.

Next, suppose that the particle in Region A moves towards the positive direction in the x-axis. Consider the reflection of this particle by the potential interface existing at $x = 0$, and the penetration of the particle into Region B.

(3) Suppose that the wave function of the injected particle is given by $\Psi_{\text{in}}(x, t) = \phi_{\text{in}}(x)e^{-i\omega t}$, choose an appropriate term for $\phi_{\text{in}}(x)$ from the two terms in the right side of Eq. (ii), and briefly describe the reason why.

(4) Explain the boundary conditions for $\phi(x)$ and $\frac{d}{dx}\phi(x)$ which must be satisfied at the interface at $x = 0$.

(5) When the energy E of the injected particle is equal to $\frac{V_0}{2}$, give an expression for C_1 using C_2. Then, choose the correct statement from the following: (a) $|C_1| > |C_2|$, (b) $|C_1| = |C_2|$, or (c) $|C_1| < |C_2|$.

(6) Under the same conditions as Question (5), and when the wave function of the particle in Region B is given by $\Psi_{\text{B}}(x, t) = \phi_{\text{B}}(x)e^{-i\omega t}$, show that $|\phi_{\text{B}}(0)|$ is larger than $|\phi_{\text{in}}(x)|$. In addition, explain the reason why this relationship does not conflict with the energy conservation law for the energy transport by this particle, in about 3 lines.

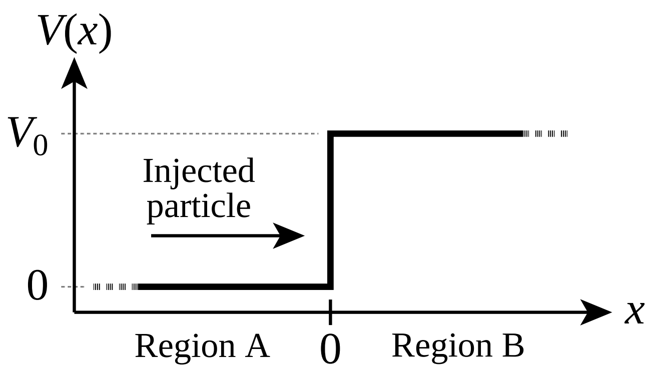

Fig. 1

II.

Consider a semiconductor, in which the bandgap E_g is 1.0 eV, the effective masses m_e and m_h of an electron in the conduction band and a hole in the valence band are $0.10m_0$ and $0.30m_0$ (m_0 is the mass of a free electron in a vacuum), respectively, and the relative dielectric constant ε_r is 10. Also suppose that impurities which become monovalent ions are doped in this semiconductor to make it a n-type semiconductor, and that such impurities create an impurity level at E_imp near the bottom of the conduction band, as shown in Fig. 2. Answer the following questions.

(1) Give the sign of charge of the impurity ionized in the semiconductor.

(2) For an absolute temperature T, the free carrier density ρ varies with respect to $1/T$, as shown in Fig. 3. For the points labeled (a) and (b) in Fig. 3, choose the most appropriate Fermi level positions from V to Z in Fig. 2, and describe the reason why, in about 3 lines each. You may omit the temperature dependence of E_g.

(3) Around the point labeled (c) in Fig. 3, the free carrier density ρ is almost independent of temperature. Describe the reason for this, in about 3 lines.

(4) In a hydrogen atom, an electron with a negative single charge is bound by an atomic nucleus with a positive single charge. In the hydrogen atomic model, the ground state energy E_1 and the Bohr radius a_B are given by

$$E_1 = -\frac{e^2}{2(4\pi\varepsilon_0)a_\mathrm{B}} \cong -14\ [\mathrm{eV}] \text{ and}$$
$$a_\mathrm{B} = \frac{\varepsilon_0 h^2}{\pi m_0 e^2} \cong 0.053\ [\mathrm{nm}],$$

where e, ε_0 and h are the elementary charge, the dielectric constant of vacuum and the Planck constant, respectively. Based on this hydrogen atomic model, obtain approximate values of the ionization energy ΔE_imp and the effective Bohr radius a_B^* for the present impurity.

(5) Choose the most appropriate point from (a) to (c) in Fig. 3 which corresponds to the situation at room temperature (300 K), and describe the reason why, using values of the Boltzman constant k_B ($\cong 1.4 \times 10^{-23}$ [J/K]) and the elementary charge e ($\cong 1.6 \times 10^{-19}$ [C]).

(6) For an impurity which becomes a divalent ion, the energy level of the impurity is not normally given based on the hydrogen atomic model in Question (4). Describe the reason for this, in about 5 lines. In addition, briefly describe which is larger, the ionization energy for the divalent ion or ΔE_imp for the monovalent ion.

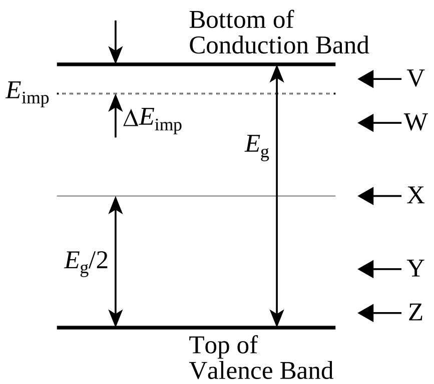

Fig. 2

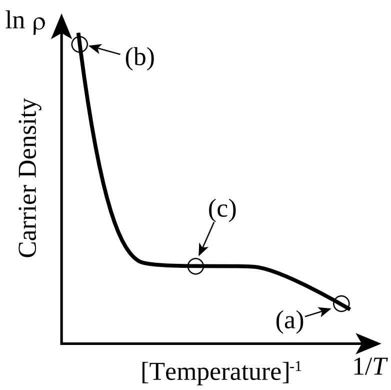

Fig. 3