Problem 6

I.

Consider a speed control system of the DC servomotor represented by Eq. (i). Here, t is time, $\omega(t)$ is the rotational angular velocity, and $u(t)$ is the control input. In addition, s is the Laplace operator. $W(s)$ and $U(s)$ are the Laplace transformations of $\omega(t)$ and $u(t)$, respectively. Answer the following questions.

$$\frac{d\omega(\mathrm{t})}{dt} = 2u(t) - 4\omega(t) \qquad \text{(i)}$$

(1) Derive the transfer function $P_0(s) = W(s) \,/\, U(s)$ of this plant.

(2) For this plant, a feedback controller $C(s)$ is designed as the proportional-integral controller expressed by Eq. (ii).

$$U(s) = C(s)\big(R(s) - W(s)\big), \qquad C(s) = K_\mathrm{p}\left(1 + \frac{1}{\tau_\mathrm{i} s}\right) \qquad \text{(ii)}$$

Here, $R(s)$ is the Laplace transformation of the speed command $r(t)$, K_p is the proportional gain, and τ_i is the integration time.

(2-i) Derive the transfer function $G(s) = W(s) \,/\, R(s)$ of the closed loop system.

(2-ii) Find the controller parameters K_p and τ_i for placing the poles of the closed loop system at -40 and -50.

(2-iii) Calculate the time response of $\omega(t)$ when a unit step function is given to the speed command $r(t)$ for the closed-loop system obtained in Question (2-ii).

(3) Draw the root locus of the closed-loop system when τ_i is fixed to 0.05 and K_p is changed from 0 to ∞ in the controller of Eq. (ii).

(4) Find K_p that gives multiple roots of the closed-loop system in Question (3).

(5) Consider the stability when the plant has a modeling error of the mechanical resonance mode that is expressed by Eq. (iii) with $0 < \zeta < 1$, for the controller $C(s)$ obtained in Question (2-ii).

$$P_1(s) = \frac{\omega_\mathrm{p}^2}{s^2 + 2\zeta\omega_\mathrm{p} s + \omega_\mathrm{p}^2} \qquad \text{(iii)}$$

(5-i) When $\omega_\mathrm{p} = 1000$ and $\zeta = 0.1$, sketch the Bode diagram of the open-loop transfer function $P_0(s)P_1(s)C(s)$ that has this modeling error using asymptotic approximations. Indicate numerical values such as the angular-frequency break-points, the slope of the gain diagram, and the angle of the phase diagram.

(5-ii) For $\omega_\mathrm{p} = 1000$, find the range of ζ that can guarantee the stability of the closed loop system.

Sample

II.

Consider a separately excited DC motor. Figure 1 shows the armature circuit. Let s be the Laplace operator. Answer the following questions.

(1) Consider deriving a mathematical model from the circuit equation and the equation of motion.

(1-i) Let $V_{\mathrm{a}}(s)$, $I(s)$, and $W(s)$ be the Laplace transformations of the terminal voltage, armature current, and rotational angular velocity, respectively. Let R and L be the armature resistance and the armature inductance, respectively. Describe the armature circuit equation in the s domain. Here the back electromotive force $V_{\mathrm{e}}(s)$ can be expressed as $K_{\mathrm{e}}W(s)$, where K_{e} is the back electromotive force coefficient.

(1-ii) Describe the equation of motion of the rotor in the s domain where J is the moment of inertia. The torque generated by the motor and the viscous friction torque are represented as $T_{\mathrm{m}}(s) = K_{\mathrm{t}}I(s)$ and $T_{\mathrm{v}}(s) = D_{\mathrm{v}}W(s)$, respectively, where K_{t} is the torque coefficient and D_{v} is the viscous friction coefficient.

(2) To what kinds of work is the input power from voltage source V_{a} converted? Explain it using equations in time domain based on the circuit model obtained in Question (1-i). You may define the necessary variables by yourself.

(3) Consider applying a current feedback control system to the motor in Question (1). Draw the block diagram. Also, show that the transfer function from the current command $I^*(s)$ to the rotational angular velocity $W(s)$ can be approximately expressed as a first-order system when the controller has a sufficiently high gain.

Sample

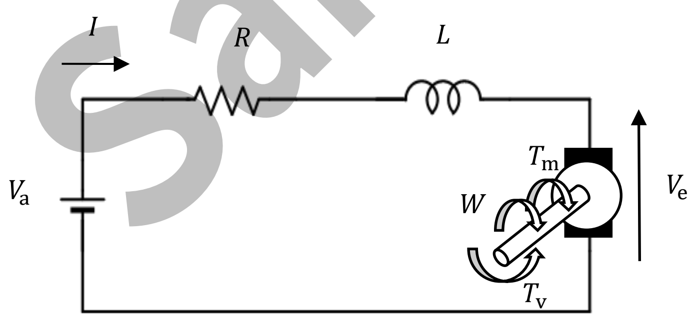

Fig. 1